Constraints on non-minimally coupled curved space electrodynamics from astrophysical observations

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Abstract

We study interactions of electro-magnetic fields with the curvature tensor of the form $\lambda R_{\mu\nu\alpha\beta} F^{\mu\nu} F^{\alpha\beta}$. Such coupling terms though are invariant under general coordinate transformation and CPT, however violate the Einstein equivalence principle. These couplings do not cause any energy dependent dispersion of photons but they exhibit birefringence. We put constraints on the coupling constant λ using results from solar system radar ranging experiments and millisecond-pulsar observations. We find that the most stringent constraint comes from pulsar observations and is given by $\lambda < 10^{11} cm^2$ obtained from the timing of binary pulsar PSR B1534+12..

1 Introduction

Einstein's general relativity is based on the twin principles of general co-ordinate invariance and the Equivalence principle. A generalization of the standard minimal coupling of electro-magnetic fields in curved space-time introduced in [1]

$$\mathcal{L} = -\frac{1}{4} \; F^{\mu\nu} \; F_{\mu\nu} + \lambda \; R^{\mu\nu\rho\sigma} \; F_{\mu\nu} F_{\rho\sigma} \tag{1}$$

is a theory which violates the Einstein's equivalence principle (which states that in a local inertial frame the theory should reduce to the form given by special relativity) as the Riemann curvature tensor does not vanish in a local inertial frame. This theory however is invariant under general coordinate transformation as well as CPT.

In this paper we study some of the experimental consequences of the curvature couplings of electro-magnetic fields and put constraints on the coupling constant λ from radar ranging past the sun [2] and from the observations of millisecond pulsars [3]. The effect of the curvature term in the lagrangian is to make the photon orbits polarization dependent. Therefore the two independent polarizations of the electromagnic waves experience different amounts of bending and time delay in a gravitational field. There is however no energy dependent dispersion of the photon trajectories.

Constraints on the dispersion relations of the generic form $p^2 - E^2 = (E/M)^n$ can be obtained from the non observation of time lag between GRB signals of different energies which have propagated over cosmological distances. Bounds of the order of $M > 10^{15} gev$ [6] have been obtained from the non-observation of an energy dependent time delay of Gamma Ray Burst signals.

An example of a coupling term which does not give rise to an energy dependent group velocity is given by $L_I = \lambda \; K^{\mu\nu\rho\sigma} \; F_{\mu\nu} F_{\rho\sigma}$ [7], where $K^{\mu\nu\rho\sigma}$ is a non-dynamical constant which has the same symmetries under exchange of indices as the Riemann tensor. This coupling violates CPT and Lorentz invariance and it gives rise to optical rotation of photon signals. Kostelecky and Mewes [7] put upper bounds $\sim 3 \times 10^{-32}$ on the components of K from the non-observation of optical activity in signals from radio galaxies.

In the present example we find that due to the Riemann term, the coupling becomes vanishingly small for cosmological background metrics and the advantage of large cosmological distances over which the effect of dispersion or polarisation is accumulated in the examples discussed earlier are lost. We find that the strongest bounds come from the consideration of compact stars. Radar echo experiments of radio signals passing in the vicinity of the sun give a bound of $\lambda < 3.9 \times 10^{19} cm^2$. This is two orders of magnitude more stringent than bounds on λ obtain from the birefringent bending of light by the sun [8]. We find that the most stringent bound on λ is obtained from the timing of binary pulsar PSR B1534+12 signals [3] and is given by $\lambda < 0.6 \times 10^{11} cm^2$.

2 Curvature coupling electrodynamics

The equation of motion for $F^{\mu\nu}$ from the Lagrangian (1) is given by

$$\nabla_\nu F^{\mu\nu} = 2\lambda \left[R^{\mu\nu\rho\sigma}(\nabla_\nu F_{\rho\sigma}) + (\nabla_\nu R^{\mu\nu\rho\sigma})F_{\rho\sigma}\right] \tag{2}$$

In addition we have the Bianchi identities

$$\nabla_\beta R^{\mu\nu}{}_{\rho\sigma} + \nabla_\sigma R^{\mu\nu}{}_{\beta\rho} + \nabla_\rho R^{\mu\nu}{}_{\sigma\beta} = 0 \tag{3}$$

Setting $\beta = \nu$ in (3) we get

$$\nabla_\nu R^{\mu\nu}{}_{\rho\sigma} = \nabla_\sigma R^{\mu}{}_{\rho} - \nabla_\rho R^{\mu}{}_{\sigma} \tag{4}$$

Using (4) in the equation of motion (2) we obtain

$$\nabla_\nu F^{\mu\nu} = 2\lambda \left(R^{\mu\nu\rho\sigma}(\nabla_\nu F_{\rho\sigma}) + (\nabla_\rho R^{\mu}_{\sigma})F^{\rho\sigma}\right) \tag{5}$$

In the absence of sources ($R^{\mu\nu} = 0$) equation (5) simplifies to

$$\nabla_\nu F^{\mu\nu} = 2\lambda \; R^{\mu\nu\rho\sigma}(\nabla_\nu F_{\rho\sigma}) \tag{6}$$

To obtain the trajectory of photons in curved space it is convenient to choose a locally flat inertial frame and derive the dispersion relations for the photon momenta $p_{(\mu)}$ in the inertial frame. The experimentally observed photon momenta in the coordinate frame k_μ is then obtained by making use of the tetrads $k_\mu = e^{(\nu)}_\mu p_{(\nu)}$. In our application the photon wavelengths will be much smaller than the curvature scale of the gravitating bodies and therefore we can make the eikonal approximation

$$\nabla_{(\mu)} F^{(\mu)(\nu)} = p_{(\mu)} \; F^{(\mu)(\nu)} \tag{7}$$

in which case the equations of motion (6) in a local inertial frame appear as

$$p_{(\nu)} \; F^{(\mu)(\nu)} = 2\lambda R^{(\mu)(\nu)(\alpha)(\beta)} p_{(\nu)} F_{(\alpha)(\beta)} \tag{8}$$

We also have the Bianchi identity,

$$p_{(\alpha)} F_{(\mu)(\nu)} + p_{(\nu)} F_{(\alpha)(\mu)} + p_{(\mu)} F_{(\nu)(\alpha)} = 0 \tag{9}$$

Setting $\mu = j$ in (8) (we denote the time index by 0 and spatial indices by Latin and four indices by Greek letters) we obtain

$$p_{(0)}F^{(j)(0)} + p_{(k)}F^{(j)(k)} - 2\lambda(R^{(j)(\nu)(\alpha)(0)}p_{(\nu)}F_{(\alpha)(0)} + R^{(j)(\nu)(\alpha)(k)}p_{(\nu)}F_{(\alpha)(k)}) = 0 \qquad (10)$$

We use the Bianchi identity (9) to write the magnetic field tensor in terms of electric field tensor

$$p_{(0)}F_{(j)(k)} = p_{(k)}F_{(j)(0)} - p_{(j)}F_{(k)(0)} \qquad (11)$$

Using (11) in (10) we finally obtain the wave equation for the electric field polarization vector in a local inertial frame,

$$(p^{(\mu)}p_{(\mu)}\delta^{(j)}_{(k)} - p^{(j)}p_{(k)} + 4\lambda R^{(j)(\nu)(\mu)}{}_{(k)}p_{(\nu)}p_{(\mu)})\ F^{(k)(0)} = 0 \qquad (12)$$

Using (8) to substitute for the second term we find that (12) reduces to the form

$$(p^{(\mu)}p_{(\mu)}\delta^{(j)}_{(k)} + 4\lambda(-\frac{p^{(j)}}{p_{(0)}}\epsilon^{(0)}{}_{(k)} + \epsilon^{(j)}{}_{(k)})F^{(k)(0)} = 0 \qquad (13)$$

where

$$\epsilon^{(\alpha)}{}_{(\beta)} \equiv R^{(\alpha)(\mu)(\nu)}{}_{(\beta)}p_{(\mu)}p_{(\nu)} \qquad (14)$$

The dispersion relations for electromagnetic waves can be obtained by setting

$$Det[p^{(\mu)}p_{(\mu)}\delta^{(j)}_{(k)} + 4\lambda(-\frac{p^{(j)}}{p_{(0)}}\epsilon^{(0)}{}_{(k)} + \epsilon^{(j)}{}_{(k)})] = 0 \qquad (15)$$

For the Schwarzschild metric

$$ds^2 = (1 - \frac{2GM}{r})dt^2 - (1 - \frac{2GM}{r})^{-1}dr^2 - r^2(d\theta^2 + Sin^2\theta d\phi^2), \qquad (16)$$

the non-zero components of Riemann tensor in a local inertial frame are

$$\begin{aligned} R_{(0)(1)(0)(1)} &= -R_{(2)(3)(2)(3)} = \frac{2GM}{r^3} \\ R_{(0)(2)(0)(2)} &= R_{(0)(3)(0)(3)} = -R_{(1)(2)(1)(2)} = -R_{(1)(3)(1)(3)} = -\frac{GM}{r^3} \end{aligned} \qquad (17)$$

and the components of $\epsilon_{(\alpha)(\beta)}$ are

$$
\begin{aligned}
\epsilon_{(0)(0)} &= \frac{GM}{r^3}(-2{p_{(1)}}^2 + {p_{(2)}}^2 + {p_{(3)}}^2) \\
\epsilon_{(1)(1)} &= -\frac{GM}{r^3}(2{p_{(0)}}^2 + {p_{(2)}}^2 + {p_{(3)}}^2) \\
\epsilon_{(2)(2)} &= \frac{GM}{r^3}({p_{(0)}}^2 - {p_{(1)}}^2 + 2{p_{(3)}}^2) \\
\epsilon_{(3)(3)} &= \frac{GM}{r^3}({p_{(0)}}^2 - {p_{(1)}}^2 + 2{p_{(2)}}^2) \\
\epsilon_{(0)(1)} &= -\frac{2GM}{r^3}p_{(0)}p_{(1)} \\
\epsilon_{(0)(2)} &= \frac{GM}{r^3}p_{(0)}p_{(2)} \\
\epsilon_{(0)(3)} &= \frac{GM}{r^3}p_{(0)}p_{(3)} \\
\epsilon_{(1)(2)} &= \frac{GM}{r^3}p_{(1)}p_{(2)} \\
\epsilon_{(1)(3)} &= \frac{GM}{r^3}p_{(1)}p_{(3)} \\
\epsilon_{(2)(3)} &= -\frac{2GM}{r^3}p_{(2)}p_{(3)}
\end{aligned} \tag{18}
$$

where we have set up the tetrad frame such that (p_1, p_2, p_3) are along $(\hat{e_r}, \hat{e_\theta}, \hat{e_\phi})$.

With these expressions we can determine the dispersion relation (15) for the Schwarzschiled geometry which can be compared with experimental observations.

3 Polarization dependent time delay

A radar signal sent across the solar system past the Sun to a planet or a spacecraft suffers an additional Shapiro time delay in general relativity which has been measured [2] and confirms Einsteins theory to about one percent accuracy. In presence of the curvature coupling terms there is an additional time delay which we calculate in the following. We consider photon trajectories tangential to the gravitating body and in the tetrad frame $p_{(\mu)} = (p_0, p_1, 0, p_3)$. The equation of motion for the (E_1, E_2) components is of the

form

$$\begin{pmatrix} p^2 + c_1(2(p_{(0)}^2 - p_{(1)}^2) + p_{(3)}^2) & 0 \\ 0 & p^2 - c_1(p_{(0)}^2 - p_{(1)}^2 + 2p_{(3)}^2) \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \end{pmatrix} = 0 \tag{19}$$

where $c_1 = 4\lambda(GM/r^3)$. Setting the determinant equal to zero yields the dispersion relations for the two propagating modes,

$$p^2 = \pm 12\lambda \frac{GM}{r^3} p_3^2 \tag{20}$$

We obtain the dispersion relation in terms of the coordinate frame 4-momentum k_μ by making the transformation $p_{(a)} = e^\nu{}_{(a)} k_\nu$. The dispersion relations in terms of k_μ is

$$\begin{aligned} (1 - 2GM/r)^{-1} k_t^2 \quad &- \quad (1 - 2GM/r) k_r^2 - (1/r^2) k_\phi^2 \\ &\mp \quad 12\lambda \frac{GM}{r^3} \frac{k_\phi^2}{r^2}) = 0 \end{aligned} \tag{21}$$

At the point of inflection of the photon trajectory $k_r = 0$ and one can evaluate the constant angular momentum p_ϕ in terms of the k_0 and the impact parameter r_0 as

$$k_\phi = r_0 k_t (1 - 2GM/r)^{-1/2} (1 \mp 12\lambda \frac{GM}{r_0^3})^{1/2} \tag{22}$$

Substituting (22) in the dispersion relation we get

$$\begin{aligned} (1 - 2GM/r)^{-1} k_t^2 \quad &- \quad (1 - 2GM/r) k_r^2 - k_t^2 \frac{r_0^2}{r^2} (1 - 2GM/r)^{-1} \\ &\mp \quad 12\lambda k_t^2 \frac{GM}{r^2 r_0} (1 - 2GM/r)^{-1} = 0 \end{aligned} \tag{23}$$

We can now solve the dispersion relations for k_t in terms of k_r to obtain,

$$k_t = \frac{k_r (1 - 2GM/r)}{(1 - \frac{r_0^2}{r^2} \frac{((1-2GM/r)}{(1-2GM/r_0)} (1 - \lambda_\pm (\frac{GM}{r0^3}))^{1/2}} \tag{24}$$

where we have defined $\lambda_\pm = \pm 12\lambda$. The four-momenta are related to the coordinates as $k^t = dt/ds$ and $k^r = dr/ds$ which enables us to obtain the photon trajectory

$$\frac{dt}{dr} = \frac{g^{tt} k_t}{g^{rr} k_r} \tag{25}$$

Using the relations (25) and (24) we obtain the relation between coordinate time t and radius r given by

$$t_f - t_i = \int_{r_i}^{r_f} \frac{dr(1-2GM/r)^{(-1)}}{(1 - \frac{r_0^2}{r^2}\frac{((1-2GM/r)}{(1-2GM/r_0)}(1 - \lambda_\pm(\frac{GM}{r0^3}))^{1/2}} \tag{26}$$

Evaluating this integral, we obtain for the time delay

$$\begin{aligned} \Delta t_{if} &= r_{if} + 2GMLog(\frac{r_f^2 + \sqrt{r_f^2 - r_0^2}}{r_i^2 + \sqrt{r_i^2 - r_0^2}}) \\ &+ \lambda_\pm \frac{GM}{r_0^2}(\frac{(r_f^2 - r_0^2)^{1/2}}{r_f} + \frac{(r_i^2 - r_0^2)^{1/2}}{r_i}) \end{aligned} \tag{27}$$

The last term is polarization dependent and therefore if one sends a pulse of unpolarized radio signal, the two orthogonal polarizations have different arrival times at the earth and we should observe a splitting of the pulse in time with the time lag between the two polarizations given by

$$\begin{aligned} t_+ - t_- &= (\lambda_+ - \lambda_-)\frac{GM}{r_0^2}(\frac{(r_f^2 - r_0^2)^{1/2}}{r_f} + \frac{(r_i^2 - r_0^2)^{1/2}}{r_i}) \\ &= \lambda\ 24\ \frac{GM_{sun}}{R_{sun}^2}(|Sin\theta_f| + |Sin\theta_i|) \end{aligned} \tag{28}$$

where θ_i is the angle between the position of the earth and $\vec{r}_0$ and θ_f is the angle between the position of the planet/spacecraft and $\vec{r}_0$. The timing measurements are accurate to $\sim 1\mu sec$. When the planet spacecraft is at superior conjunction ($Sin\theta_i = Sin\theta_f = 1$)and taking $M_{sun} = 1.98 \times 10^{33} gm$ and $R_{sun} = 6.95 \times 10^5 km$, the lack of pulse splitting of the observed radar signals to within a μsec accuracy translates to a bound

$$\lambda < 1.1 \times 10^{20} cm^2. \tag{29}$$

This bound is about three orders of magnitude more stringent than the one obtained in [8] from the analysis of bending of light by the gravitational field of the sun in non-minimally coupled electro-magnetism.

Since the time is proportional to GM/r^2 this effect is largest for compact stars where ($M \sim M_\odot$ and $R \sim 10km$). Observation of the time delay in

binary pulsar PSR B1534+12 [3] yields a value of $6.3 \pm 1.3 \mu sec$ for the Shapiro time delay by the binary companion which has a mass of $1.33 M_{\odot}$ and radius of $10km$. The theoretical estimate [4] of this time delay assuming general relativity to be the correct theory is $6.6\mu sec$. Taking an upper limit of $1\mu sec$ as the maximum contribution of the polarization dependent correction to the GR time delay we have an get an upper bound on λ ,

$$\lambda < 0.6 \times 10^{11} cm^2 \tag{30}$$

This bound is far more stringent than those from the solar ranging experiments. Note that the constraint on λ from pulsars is obtained by taking the difference of the travel times of the radio signals with orthogonal polarizations. There are systematic uncertainties in the measurement of total travel time from pulsars as discussed in [5]. However these systematic effects like the emission time, pulsar velocity, earths velocity and position etc are independent of the polarization of the radio signal so these uncertainties drop out when we take the difference of travel time between two different polarizations.

For radial photons the four momentum in the inertial frame is $p_{(\mu)} = (p_0, p_1, 0, 0)$ and only non-zero components of the coordinate frame four momentum are $k_\mu = (k_t, k_r, 0, 0)$. The determinant condition on radial propagation gives $p^2 = 0$ and the non-minimal coupling terms have no effect on radially propagating electromagnetic waves.

4 Conclusions

We find that the curvature coupling of electromagnetic fields give rise to dispersion free photon propagation, where the photon phase and group velocities are dependent on the polarization. The best constraints on such couplings can be obtained from the timing of electromagnetic waves in the vicinity of compact objects. The curvature coupling situation is different from the types of couplings where the effect accumulates over the distance of propagation and stringent bounds on extra equivalence principle violating couplings are put from observations of radio galaxies [7] or gamma-ray bursts [6].

In this paper we have taken the view that the classical lagrangian can possibly have EEP violating but general covariant terms with large coefficients which we bound from observations. If these curvature couplings arise as only

as loop corrections to Einteins general relativity [9] and if $\lambda \sim {M_p}^{-2}$ then there are so far no realistic means of observing such couplings in experiments.

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